* **Gaussian Elimination Method**

1. Reduction to upper-triangular form
2. Backward substitution

The Gaussian elimination method fails if any one of the pivot elements becomes zero. In such a situation, we rewrite the equations in a different order to avoid zero pivots. Changing the order of equations is called **pivoting**.

* **Gauss-Jordon Elimination Method**

Partial pivoting is also used whenever the pivot element becomes zero.

Example

* **Crout’s Reduction Method**

1. Decompose into and .
2. Solve for .
3. Solve for .

In this method, the coefficient matrix of the system of equations

is decomposed into the product of two matrices and .

Where, is an *upper-triangular matrix* and is a *lower-triangular matrix* with 1’s on its main diagonal.

is the same as the coefficient matrix at the end of forward elimination step.

is obtained using the multipliers that were used in the forward elimination process.

* **Jacobi’s Method**

Example

Find the solution to the following system of equations using Jacobi’s iterative method. Taking the initial starting of solution vector as

* **Gauss-Seidel Iteration Method**

Example

Find the solution of the following system of equations using Gauss-Seidel method. Taking the initial starting of solution vector as

* **Matrix Inversion**

The matrix is called the invers of . A matrix without an inverse is called **singular** (or **noninvertible**).

For any nonsingular matrix ,

* + is unique.
  + is nonsingular and
  + If is also a nonsingular matrix, then
* **Gaussian Elimination Method**
* **Gauss-Jordan Method**
* **Eigenvalues and Eigenvectors**

Let be an square matrix. Suppose, there exists a scalar and a vector

Such that

Then is the **eigenvalue** and is the corresponding **eigenvector** of the matrix .

* + If is a triangular matrix -upper, lower or diagonal, the eigenvalues of are the diagonal of .
  + is an eigenvalue of if is a singular (noninvertible) matrix.
  + and have the same eigenvalues.
  + is the product of the absolute values of the eigenvalues of .
* **Power Method**

It is used to find the largest eigenvalue in an absolute sense.

1. Choose the initial vector such that the largest element is unity (or 1).
2. The normalized vector is pre-multiplied by matrix .
3. The resultant vector is again normalized.
4. This process of iteration is continued, and the new normalized vector is repeatedly pre-multiplied by the matrix until the required accuracy is obtained.

Example

Find the eigenvalue of largest modulus, and the associated eigenvector of the matrix. Choose an initial vector as

* **The Absolute Relative Approximate Error**
* **Forward Differences**

For a given table of values

with equally spaced abscissas of a function we define the forward difference operator as follows

To be explicit, we write

These differences are called *first differences* of the function and are denoted by the symbol .

Similarly, the differences of the *first differences* are called *second differences*, defined by

Thus, in general

* **Backward Differences**

For a given table of values

of a function with equally spaced abscissas, the first backward differences are usually expressed in terms of the backward difference operator as

To be explicit, we write

In general

* **Central Differences**

We use the symbol to represent central difference operator and the subscript of for any difference as the average of the subscripts.

* **Shift Operator,**

Let be a function of , and let takes the consecutive values , , , etc.

We then define an operator having the property

Thus, when operates on , the result is the next value of the function.

* **Average Operator,**
* **Differential Operator,**
* **Few Results Using , , , , and**

From the definition of operator and , we have

From the definition of operator and , we have

From the definition of operator and , we have

From the definition of operators and , we have

Using Taylor series expansion, we have

Example: Prove that

* **Newton’s Forward Difference Interpolation**

This formula is also known as Newton-Gregory forward difference interpolation formula.

* **Newton’s Backward Difference Interpolation**
* **Lagrange’s Interpolation Formula**
* **Divided Differences**
* **Newton’s Divided Difference Interpolation**